Turbulent behaviour in magnetic hydrodynamics is not universal

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Abstract

A short distance expansion method (SDE) that is well known in the quantum field theory for analysis of turbulent behaviour of stochastic magnetic hydrodynamics of incompressible conductive fluid is applied. As a result is shown that in an inertial range the turbulent spectra of magnetic hydrodynamics depend on a scale of arising of curls.

1 Introduction

The methods of quantum field theory are successfully applied for description of the critical phenomena and developed turbulence recently. This approach has an important advantage before the classical one. For example it enables one to renormalize all the correlation functions of a model as well as to define their various asymptotics.

A stochastic problem of magnetic hydrodynamics (MHD) of incompressible conductive fluid with external random force f^{φ} and rotor of random current f^{θ}

$$\mathcal{D}_t v_i = \nu \Delta v_i - \partial_i p + (\theta\partial)\theta_i + f_i^v, \quad \mathcal{D}_t = \partial_t + v\partial, \tag{1}$$
$$\mathcal{D}_t \theta_i = \nu' \Delta \theta_i + (\theta\partial) v_i + f_i^{\theta}$$

is equivalent to a theory of four fields with generating functional of renormalized correlation functions (Green functions)

$$G(A_\phi) = \int \mathcal{D}\Phi \det M \exp(S_R(\Phi) + \Phi A_\phi), \quad \Phi = \{\varphi, \varphi', \theta, \theta'\}$$

in which the renormalized functional of action is

$$S_R = \frac{1}{2} g_1 \nu^3 M^{2\varepsilon} \varphi' D^{\varphi\varphi} \varphi' + \frac{1}{2} g_2 \nu^3 M^{2a\varepsilon} \theta' D^{\theta\theta} \theta' + g_3 \nu^3 M^{(1+a)\varepsilon} \varphi' D^{\varphi\theta} \theta' + \qquad (2)$$

$$+\varphi'[-\partial_t \varphi + Z_1 \nu \Delta\varphi - (\varphi\partial)\varphi + Z_3(\theta\partial)\theta] + \theta'[-\partial_t\theta + Z_2 \nu u \Delta\theta - (\varphi\partial)\theta + (\theta\partial)\varphi]$$

(necessary integration on $\mathbf{x}$ and t as well as summation on repeated badges are implied). Here the fields $\varphi(\mathbf{x}, t)$ and $\theta(\mathbf{x}, t)$ are both transversal (the vector field of velocity φ due to incompressibility of fluid ($\partial_i \varphi_i = 0$) and the pseudovector field θ as connected with transversal field of a magnetic induction $\mathbf{B} : \theta = \mathbf{B}/\sqrt{4\pi\varrho}$ (ϱ is a density of fluid, ν is a viscosity, p is a pressure). We shall use the dimensionless constant λ (it means inverted number of Prandtl) instead of $\nu' = c^2/4\pi\sigma$ (here σ is a conductivity, c is the velocity of light) by this way $\nu' = \lambda\nu$.

Under construction of the model existence of an inertial range is implied therein a real external energy pumping (correlators of the random forces) can be simulated by exponential model of δ-function.

$$D_{is}^{vv} = g_{10} \nu_0^3 P_{is} d_{vv}, \quad D_{is}^{\theta\theta} = g_{20} \nu_0^3 P_{is} d_{\theta\theta}, \quad D_{is}^{v\theta} = g_{30} \nu_0^3 \varepsilon_{ism} k_m d_{v\theta}, \qquad (3)$$

$$d_{vv} = k^{4-d-2\varepsilon}, \quad d_{\theta\theta} = k^{4-d-2a\varepsilon}, \quad d_{v\theta} = k^{3-d-(1+a)\varepsilon}.$$

In momentum-frequency representation (from frequency the correlators D_{is} do not depend) P_{is} is the transversal projector, d means the dimension of space (completely antisymmetric pseudotensor ε_{ism} is determined only at $d = 3$).

The factors g_0 in correlators are played a role of charges; the positive constant a is an arbitrary parameter. The parameter ε serves for construction of decomposition of correlation functions, as the physical value of ε the $\varepsilon_p = 2$ is considered that simulates energy pumping from the large-scale movements of fluid.

The model (1 -3) was investigated in [1] in the first order on ε with the help of recursive renormalization group method. It is shown that in the system two different regimes of developed turbulence can be realized: "kinetic" and "magnetic" that are connected with existence of two infrared (IR) stable fixed points of RG transformation. In [1] the critical asymptotics of spectral density of energy $< \varphi(\mathbf{k})\varphi(-\mathbf{k}) >$ and $< \theta(\mathbf{k})\theta(-\mathbf{k}) >$ were determined. In [2] more general formulation of the problem with inclusion of the cross correlator of random force $D^{\varphi\theta}$ was considered by the quantum field RG method. The renormalization constants Z_i were calculated there in the first order of g_α:

$$Z_1 = 1 - \frac{g_1 d(d-1)}{4B\varepsilon} - \frac{g_2(d^2+d-4)}{4Ba\lambda^2\varepsilon}, \quad Z_3 = 1 + \frac{g_1}{B\lambda\varepsilon} - \frac{g_2}{Ba\lambda^2\varepsilon},$$

$$Z_2 = 1 - \frac{g_1(d+2)(d-1)}{2B\lambda(\lambda+1)\varepsilon} - \frac{g_2(d+2)(d-3)}{2Ba\lambda^2(\lambda+1)\varepsilon};$$

here $B = d(d+2)(4\pi)^{d/2}\Gamma(d/2)$ and $\Gamma(x)$ is the gamma-function.

Renormalizability of the model was proven, and situation in the charging space of IR-stable fixed points corresponding to kinetic and magnetic regimes were found:

$$g'_{1*} \equiv \frac{g_{1*}}{B\lambda_*} = \frac{\varepsilon(1+\lambda_*)}{15}, \quad g_{2*} = 0, \quad \lambda_* = \frac{\sqrt{43/3}-1}{2}, \tag{4}$$

(the region of stability of the point is $a < 1.16$.)

$$g_{1*} = \lambda_* = 0, \quad g'_{2*} \equiv \frac{g_{2*}}{B{\lambda_*}^2} = a\varepsilon \tag{5}$$

(this one is stable at $a \geq 0.25$). In paper [3] features of a scaling behaviour in the model were investigated, and critical dimensions of fields and parameters of the theory are found out in the both critical regimes.

We shall be interested in dependence of characteristics of developed turbulence (the correlation functions) in the inertial range from conditions of arising of large-scale curls. In the theory of developed turbulence it is supposed, that the energy pumping into the inertial range is executed by the vortices of a large size Λ. We shall take into account this scale having supplied the model (1 -3) by an infrared mass parameter $m \equiv 1/\Lambda$ to consider the relative corrections to developed turbulent spectra with the help of SDE method.

According to this method following the operator decomposition is fair:

$$\phi(\mathbf{x_1}, t)\phi(\mathbf{x_2}, t) \simeq \sum_i c_i(\mathbf{r})F_i(\mathbf{x}, t); \tag{6}$$

where $x \equiv (x_1 + x_2)/2$, $r \equiv x_1 - x_2$, F_i are various local averages (composite operators). The averaging of (6) yields the asymptotics for pair correlation functions at $mr \to 0$

$$< \phi(\mathbf{x_1}, t)\phi(\mathbf{x_2}, t) > \simeq \sum_i c_i(\mathbf{r})a_i m^{\Delta_{F_i}}; \tag{7}$$

here Δ_{F_i} are the critical dimensions of the composite operators; a_i means some constants. On decomposition (7) we conclude, that from the point of view of an opportunity of transition to a massless theory ($m \to 0$) in the inertial range the operators with a negative critical dimension are dangerous.

2 Critical dimensions of the composite operators in the model of magnetic hydrodynamics

The renormalized operators are defined by the formula $F_i = Z_{ik}F_k^R(\Phi_R)$. On the known matrix Z_{ik} a matrix of anomalous dimensions $\gamma_{ik} = (Z^{-1})_{ij}\mathcal{D}_M Z_{jk}$ is calculated and then the matrix of critical dimensions $\Delta_{ik} = (d_F^k)_{ik} + \Delta_\omega(d_F^\omega)_{ik} + \gamma_{ik}$ where Δ_ω designates critical dimension of frequency, and d_F^k, d_F^ω are the momentum and frequency canonical dimensions of F; they are being determined from requirement of frequency and momentum dimensionless of terms of the action functional.

The particular critical dimensions are eigenvalues of the matrix Δ_{ik}. They correspond to linear combinations of composite operators $L_i(F^R) = U_{ik}F_k^R$ which diagonalize the matrix Δ_{ik}.

We shall consider the dimensions of elementary composite operators of the MHD model: $\phi_i\phi_k$, $\phi'\phi$, as well as vector operators $(\partial\phi\phi)_i$ with various transpositions of badges.

The tensor $\phi_i\phi_k$ is a sum of two independent tensors

$$\frac{1}{d}\phi^2\delta_{ik}, \qquad \phi_i\phi_k - \frac{1}{d}\phi^2\delta_{ik}. \tag{8}$$

Convolution of the first operator on badges yields family of scalar operators $\phi\phi$

$$F_1 = \frac{1}{2}v_iv_i; \quad F_2 = \frac{1}{2}\theta_i\theta_i; \quad F_3 = \theta_iv_i.$$

Trace of the second expression in (8) is equal to zero.

Essential property of the theory (1 - 3) is Galileian invariancy; it was used in [4] for investigation of the composite operators in a problem of usual stochastic hydrodynamics. A non-stationary Galileian transformations of the fields

$$\varphi_a(\mathbf{x},t) = \varphi(\mathbf{x}+\mathbf{u}(t),t) - \mathbf{a}(t); \quad \varphi'_a(\mathbf{x},t) = \varphi'(\mathbf{x}+\mathbf{u}(t),t);$$
$$\theta_a(\mathbf{x},t) = \theta(\mathbf{x}+\mathbf{u}(t),t); \quad \theta'_a(\mathbf{x},t) = \theta'(\mathbf{x}+\mathbf{u}(t),t);$$

(a parameter of transformation - $\mathbf{v}(t)$ is the vector function dependent only from a time well decreasing at $|t| \to \infty$ and $\mathbf{u}(t) = \int_{-\infty}^{t} dt\mathbf{v}(t')$) realized in the Ward identity leads [5] to

$$\int dx\{a_{0\alpha}[\frac{\partial F_\alpha}{\partial\varphi_s} - \frac{\partial F_\alpha}{\partial(\partial_t\varphi_k)}\partial_s\varphi_k] + \partial_t[a_{0\alpha}\frac{\partial F_\alpha}{\partial(\partial_s\varphi_k)}]\} < \infty. \tag{9}$$

Here $a_{0\alpha}$ are the nonrenormalized functions of sources of family of operators F_α, and symbol "$< \infty$ " means that the considered functional is finite.

We shall assign $G_1 = v_iv_k/2$ and $G_2 = \theta_i\theta_k/2$. The operator $G_3 = v_i\theta_k$ is pseudotensor so it doesn't mix with the first two operators due to renormalization. G_1 is noninvariant of the Galileian transformations so the formula (9) permits to prove that G_1 is finite ($Z_{11} = 1$), and that it doesn't mix to the operator G_2 in proceeding of renormalization, hence $Z_{21} = 0$ (one can say, that the operator G_1 aren't being renormalized and doesn't mix to the Galileian invariant operator G_2 since G_1 is not Galileian invariant). Besides, from (9) the absence of divergences in diagrams with the composite operator G_3 (it is noninvariant of Galileian transformations also) is followed. Then from the definition of renormalized composite operators it is easy to get the expression: $Z_{33} = Z_{v_i\theta_k} = Z_\theta^{-1}$.

Thus, the matrix of renormalization constants $\mathbf{Z}_{\alpha\delta}$ of the operators G_i is

$$\mathbf{Z}_{\alpha\beta} = \begin{pmatrix} 1 & Z_{12} & 0 \\ 0 & Z_{22} & 0 \\ 0 & 0 & Z_\theta^{-1} \end{pmatrix}.$$

Unknown elements of the matrix one can calculate with the help of standard diagrams with the following propagators

$$< \varphi_i(\mathbf{k},t)\varphi'_j(-\mathbf{k},0) >_0 = P_{ij} e^{-\nu k^2 t}\theta(t);$$
$$< \varphi_i(\mathbf{k},t)\varphi_j(-\mathbf{k},0) >_0 = \frac{1}{2} P_{ij} g \nu^2 k^{2-d-2\varepsilon} M^{2\varepsilon} e^{-\nu k^2 |t|};$$
$$< \theta_i(\mathbf{k},t)\theta'_j(-\mathbf{k},0) >_0 = P_{ij}\theta(t) e^{-\nu\lambda k^2 t};$$
$$< \theta_i(\mathbf{k},t)\theta_j(-\mathbf{k},0) >_0 = \frac{1}{2} P_{ij} \frac{g' \nu^2 k^{2-d-2a\varepsilon}}{\lambda} M^{2a\varepsilon} e^{-\nu\lambda k^2 |t|}; \qquad (10)$$
$$< \theta_i(\mathbf{k},t)\varphi_j(-\mathbf{k},0) >_0 = \varepsilon_{isj} \frac{g'' \nu^2 k^{1-d-(1+a)\varepsilon}}{(1+\lambda)} M^{(1+a)\varepsilon} k_s [e^{-\lambda\nu k^2 t}\theta(t) + e^{\nu k^2 t}\theta(-t)];$$

Account of the renormalization constants for the scalar operator we execute having curtailed the badges δ_{ik}.

The calculations in one-loop approximation under diagrams specified on a fig. 1 of the appropriate constants $Z_{\alpha\delta}$ in the theory (2) yelds a renormalized action for the generating functional of correlation functions with the composite operators F_α: $\bar{S}_R = S_R + a_{0\alpha} Z_{\alpha\delta} F^R_\delta(Z_\phi \phi)$; here the renormalization constants are

$$Z^{\varphi\theta}{}_{12} = -\frac{\lambda(d+2)(d-1)}{2(\lambda+1)}\left(\frac{g'_1}{\varepsilon} - \frac{g'_2}{a\varepsilon}\right); \quad Z^{\theta^2}{}_{22} Z^2_\theta = 1 + \frac{(d-1)(d+2)}{2(\lambda+1)}\left(\frac{g'_1}{\varepsilon} - \frac{g'_2}{a\varepsilon}\right);$$
$$Z^{\varphi_i\theta_j}{}_{12} = -\frac{\lambda}{2(\lambda+1)}\left(\frac{g'_1}{\varepsilon} - \frac{g'_2}{a\varepsilon}\right), \quad Z^{\theta_i\theta_j}{}_{22} Z^2_\theta = 1 + \frac{1}{2(\lambda+1)}\left(\frac{g'_1}{\varepsilon} - \frac{g'_2}{a\varepsilon}\right). \qquad (11)$$

As far as in the fixed points of RG transformation some values of the charges approach to zero in avoidance of trivialization of asymptotics in all orders of the ε-decomposition it is useful to redefine the fields as follows:

$$\theta \to \sqrt{g_2\nu^3} M^{a\varepsilon}\theta; \qquad v \to \sqrt{g_1\nu^3} M^{\varepsilon} v; \qquad (12)$$
$$\theta' \to \theta' / \sqrt{g_2\nu^3} M^{a\varepsilon}; \qquad \varphi' \to \varphi' / \sqrt{g_1\nu^3} M^{\varepsilon}.$$

The transformations don't change positions of the fixed points and values of the renormalization constants (the canonical dimensions of the operators F_α vary only).

The critical dimensions of the operators G_i calculated on the constants $Z_{\alpha\alpha}$ in the first order of ε and at any d are listed in table I.

It should be noticed that the value of Δ_1 is exact. The constant Z_{12} defines an admixture to G_1 of the operator G_2. Considering (2) in a condition of the kinetic regime with the fields of (12) we have $G_2 \to g_2 G_2$ and taking into account that in the kinetic fixed point (4) $g^*_2 = 0$ it's easy to show that just the operators G_1, G_2 (instead of their mixture) have the dimension Δ_i in this point.

The constant $Z_{12} \sim O(\lambda)$ which is responsible for mixing of the operators disappears in the magnetic point.

We shall consider elements of a matrix Z_{ik} correspond to renormalization of the set of scalar and vector composite operators $\phi'\phi$ and $(\partial\phi\phi)_i$.

At this set there are the operators reducing to a total differential $\partial(\phi\phi)$ with various transpositions of badges. Renormalization of them is equivalent to renormalization of the operators $\phi\phi$ have considered above. The critical dimensions is being appropriated to these operators surpass dimensions located in table I per unit of.

The operators of a $\phi'\phi$-type don't mix to any other operators and aren't being renormalized because of 1-irreducible diagrams that is responsible for mixing of these operators with the other are equal to zero as far as they contain cycles of advancing lines. A structure of interactions in the (2) provides removal from each diagram a one derivative on each external line of a ϕ'-type that effectively lowers an index of divergence of the diagrams. Therefore, in the minimal subtraction (MS) scheme of renormalization that is appropriate to these operators the diagonal elements are $Z_{\alpha\alpha} = 1$, and all nondiagonal ones are equal to zero. Thus, $\Delta_{\phi'\phi} = 3$.

The remaining vector operators $F_1 = v_i v^2$ and $F_2 = v\theta^2$ are true tensors, and $F_3 = \theta_i\theta^2, \quad F_4 = \theta v^2$ are pseudotensors; these pairs of the operators are being renormalized independently from each other. The operators F_{1-2} are noninvariant to Galileian transformations, thereof, it is easy to approve [5] the finiteness of the operator v^3 as well as that it doesn't mix to $v\theta^2$ due to renormalization. Thus, $Z_{11} = 1, \quad Z_{21} = 0$.

Similarly, the Galileian invariant operator F_3 can't mix to the operator F_4 that means $Z_{34} = 0$.

For definition of remaining elements of the matrix Z_{ik} in a one-loop approximation it is necessary to consider the diagrams that is shown on a fig. 2. It gives

$$Z_{12} = -3\frac{\lambda(d+2)(d-1)}{(\lambda+1)}(\frac{g_1'}{\varepsilon} - \frac{g_2'}{a\varepsilon}), \quad Z_{33}Z_\theta^3 = 1 + 3\frac{(d+2)(d-1)}{(\lambda+1)}(\frac{g_1'}{\varepsilon} - \frac{g_2'}{a\varepsilon}),$$

$$Z_{22}Z_\theta^2 = 1 + \left(\frac{(d-1)(d+2)}{(\lambda+1)} - 1\right)(\frac{g_1'}{\varepsilon} - \frac{g_2'}{a\varepsilon}), \tag{13}$$

$$Z_{43}Z_\theta = -\frac{\lambda(d+2)(d-1)}{(\lambda+1)}(\frac{g_1'}{\varepsilon} - \frac{g_2'}{a\varepsilon}), \quad Z_{44}Z_\theta = 1 - \frac{g_1'}{\varepsilon} + \frac{g_2'}{a\varepsilon},$$

The matrix of renormalization constants of this family of the operators has a block triangular form, so the critical dimensions are determined by the diagonal elements $Z_{\alpha\alpha}$.

The values of critical dimensions calculated on (13) are listed in table 2.

The nondiagonal elements Z_{12} and Z_{43} define an admixture of the operators F_1 and F_4 to F_2 and F_3. Taking into account that in the kinetic mode $F_1 = {g_1}^{3/2}v^3$, $F_2 = g_1^{1/2}g_2 v\theta^2$ $F_3 = {g_2}^{3/2}\theta^3$, and $F_4 = g_1{g_2}^{1/2}\theta v^2$ and that $g_{2*} = 0$ in the fixed point (5) it is possible to assert the certain critical dimensions are belonged to the

operators, instead of their linear combinations. In the magnetic point (6) $Z_{12} = Z_{34} = 0$.

3 Discussion of the results

It is important to note that any of the operators considered can't participate as amendments for phenomenological equations of MHD. The operators are being possessed of the essential critical dimensions don't contain auxiliary fields ϕ', but the operators of a $\phi'\phi$-type (a function of response) are inessential and don't satisfy the requirements of Galileian invariancy also.

The operators of canonical dimension $d = 3$ define new nonanalytic corrections to the spectra of developed turbulence that was found in [3]. According to SDE method such corrections for a pair correlation function $<\phi_1\phi_2>$ can be represented as follows:

$$< \phi_1(\mathbf{k}, t)\phi_2(-\mathbf{k}, t) >= Ak^{-d-\Delta_{\phi_1}-\Delta_{\phi_2}} \left(1 + \sum_i b_i \left(\frac{m}{k}\right)^{\Delta_{F_i}}\right); \tag{14}$$

(here Δ_ϕ are the critical dimensions of the fields).

As far as in the inertial range the following estimation is correct $m/k \ll 1$, the formula (14) results to nonanalyticities in a case of $\Delta_{F_i} < 0$. At the real value of $\varepsilon = 2$ the critical dimensions of G_2 $(a > 1/2$, F_1, F_2 $(a > 0.243)$, F_3, F_4 $(a > 0.82)$ in the kinetic point become negative. The dimension of F_4 at $a > 3/4$ is negative in the magnetic point also.

Thanking to Galileian invariancy of the theory we can refer to the results of [4] where the terms of the sum of (14) for static correlation functions connected with the Galileian noninvariant composite operators were proven to yield not a contribution. Those in our case are F_1, F_2, and F_4. They can participate in the decomposition (13) only for dynamic correlation functions. The Galileian invariant operator F_3 gives the contribution in the sum in all the cases.

Comparing of the dimensions of the composite operators of families $\phi\phi$ and $(\partial\phi\phi)_i$ at the real value $\varepsilon = 2$ one can see that the set $(\partial\phi\phi)_i$ appears more essential in the scaling range (so in the kinetic regime $\Delta_{\theta^2} > \Delta_{v\theta^2}, \Delta_{\theta^3}$, $\Delta_{v^2} > \Delta_{v^3}$). One can assume that the tendency of growth of essentiality of operators is being demonstrated by the elementary operators in the model of magnetic hydrodynamics will be saved for more complex operators.

The results received by us testify that in the inertial range in MHD the correlation functions depend on the external integral turbulent scale m. Thus, the behaviour in the model is not universal.

This important result can be checked experimentally besides the particular values of critical dimensions of the composite operators calculated here can be measured on an experiment too that will be served certainly to check of the offered theory.

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Table I

The critical dimensions of the composite operators of $\phi_i\phi_j$ type		
Operator	Kinetic point	Magnetic point
$(G_1)_{ij}$	$2-4/3\varepsilon$	2
$(G_2)_{ij}$	$2-2(a-3/10)\varepsilon$	$2+3a\varepsilon$
$(G_3)_{ij}$	$2-(a+1/3)\varepsilon$	$2+a\varepsilon$
$(G_1)_{ij}\delta_{ij}$	$2-4/3\varepsilon$	2
$(G_2)_{ij}\delta_{ij}$	$2-2a\varepsilon$	$2+12a\varepsilon$
$(G_3)_{ij}\delta_{ij}$	$2-(a+1/3)\varepsilon$	$2+a\varepsilon$

Table II

The critical dimensions of the composite operators of $(\partial\phi\phi)_i$ type		
Operator	Kinetic point	Magnetic point
F_1	$3-2\varepsilon$	3
F_2	$3-2(a+0.507)\varepsilon$	$3+18a\varepsilon$
F_3	$3-3(a+1)\varepsilon$	$3+60a\varepsilon$
F_4	$3-(a+0.68)\varepsilon$	$3-2a\varepsilon$

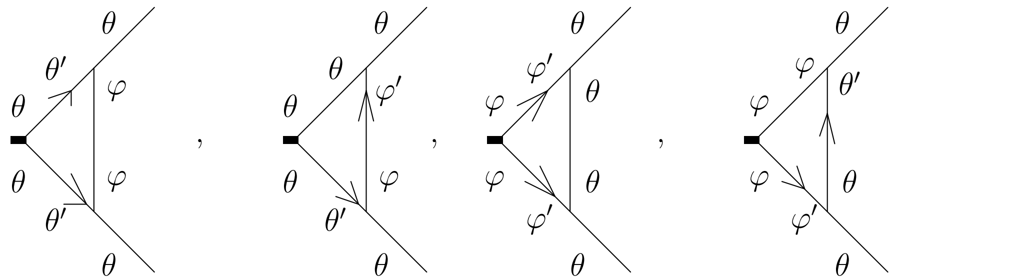

fig. 1 The set of one-loop 1-irreducible diagrams are responsible for the nontrival renormalization constances of the composite operators of a $\phi_i\phi_j$ type.

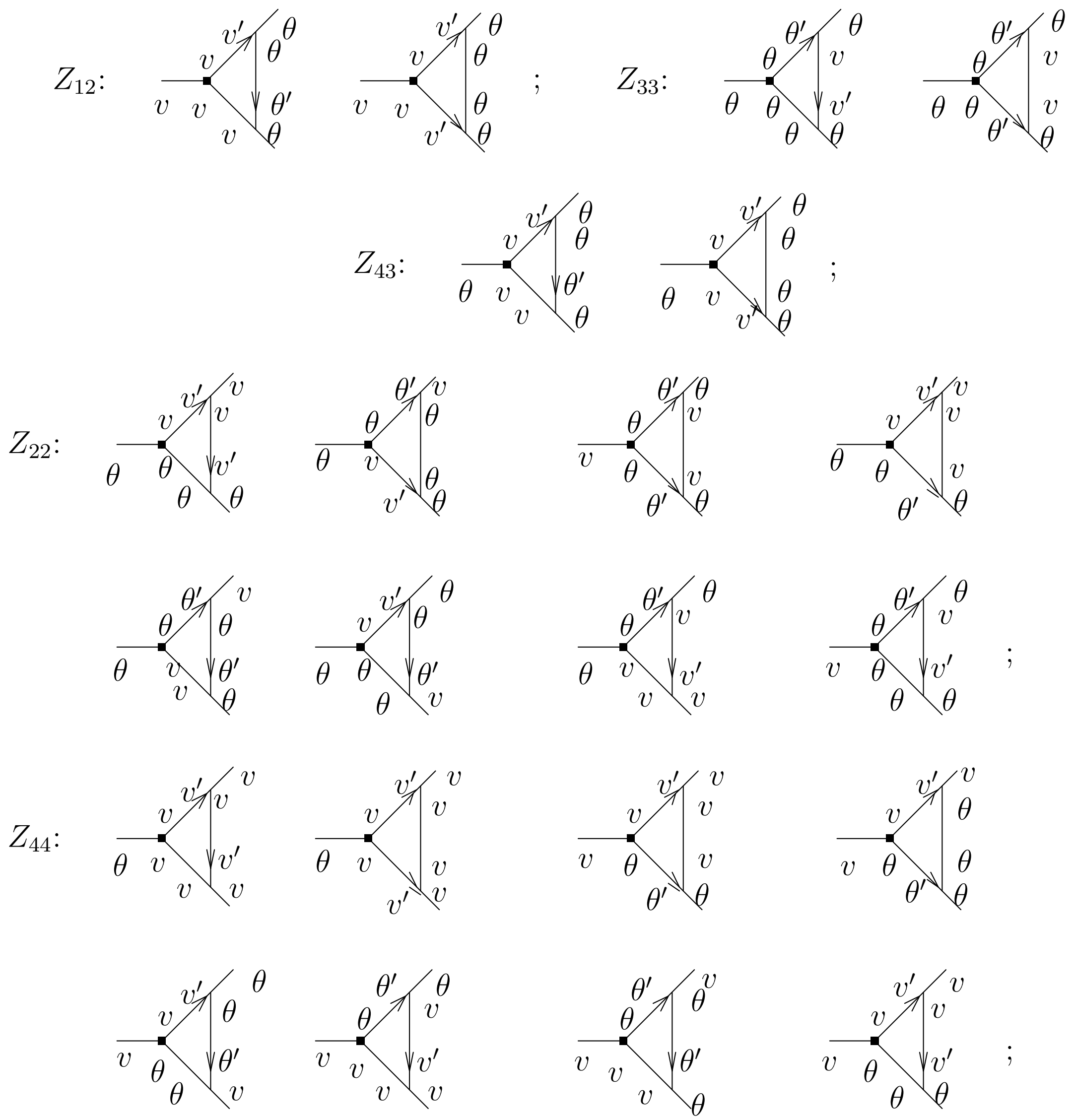

fig. 2 The set of one-loop 1-irreducible diagrams are responsible for the nontrival renormalization constances of the composite operators of a $(\partial\phi\phi)_i$ type.